1. Lecture 9:

• Last time, we initiated our study of 3D $\mathcal{N}=2$ SUSY and discussed the following representations

• The chiral multiplet ($\bar{\mathcal{D}}_\alpha\Phi=0$), which is a function of θ_α and $y^\mu=x^\mu-i\theta\gamma^\mu\bar\theta$

$$\begin{aligned}\Phi &= \phi(y)+\sqrt{2}\theta\psi(y)+\theta^2F(y)=\phi(x)-i\theta\gamma^\mu\bar\theta\partial_\mu\phi(x)-\frac{1}{4}\theta^2\bar\theta^2\partial^2\phi(x)+\sqrt{2}\theta\psi(x)\\ &+ \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\gamma^\mu\bar\theta+\theta^2F(x)\ .\end{aligned} \tag{1.1}$$

Similarly, we have an anti-chiral multiplet ($\mathcal{D}_\alpha\bar\Phi=0$)

$$\begin{aligned}\bar\Phi &= \bar\phi(y)-\sqrt{2}\bar\theta\bar\psi(y)-\bar\theta^2\bar F(y)=\bar\phi(x)+i\theta\gamma^\mu\bar\theta\partial_\mu\bar\phi(x)-\frac{1}{4}\theta^2\bar\theta^2\partial^2\bar\phi(x)-\sqrt{2}\bar\theta\bar\psi(x)\\ &- \frac{i}{\sqrt{2}}\bar\theta^2\theta\gamma^\mu\partial_\mu\bar\psi(x)+\bar\theta^2\bar F(x)\ .\end{aligned} \tag{1.2}$$

• We also found a (say $U(1)$) vector multiplet (in the WZ gauge)

$$V=\theta\gamma^\mu\bar\theta A_\mu-i\bar\theta\theta\sigma-i\theta^2\cdot\bar\theta\bar\lambda+i\bar\theta^2\theta\lambda-\frac{1}{2}\theta^2\bar\theta^2D\ , \tag{1.3}$$

which transforms as follows under gauge transformations

$$V\to V+\frac{i}{2}(\Lambda-\bar\Lambda)\ , \tag{1.4}$$

with Λ chiral and $\bar\Lambda$ anti-chiral.

• To get a field-strength, we want a gauge invariant superfield. Clearly, we get this from setting

$$\Sigma=\frac{i}{2}D\bar DV=\frac{i}{2}\bar DDV\ , \tag{1.5}$$

since, *as you showed on the homework*, $\mathcal{D}\bar{\mathcal{D}}\Lambda=\bar{\mathcal{D}}\mathcal{D}\bar\Lambda=0$. This can also be shown by contracting

$$\epsilon^{\alpha\beta}\left\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_\beta\right\}=\epsilon^{\alpha\beta}(-2i\gamma^\mu_{\alpha\beta}P_\mu)=0\ . \tag{1.6}$$

• Note that since $\mathcal{D}\bar{\mathcal{D}}=\bar{\mathcal{D}}\mathcal{D}$, we have

$$\mathcal{D}^2\Sigma=\bar{\mathcal{D}}^2\Sigma=0\ . \tag{1.7}$$

These are the equations satisfied by a conserved current superfield, with a conserved current as the spin one component at $\mathcal{O}(\theta\bar\theta)$... A SUSY generalization of a conserved current...

• Indeed, we have

$$\Sigma = \sigma - \theta\bar{\lambda} + \bar{\theta}\lambda + \frac{1}{2}\theta\gamma^{\mu}\bar{\theta}\epsilon_{\mu\nu\rho}F^{\nu\rho} - i\theta\bar{\theta}D + \cdots \ , \tag{1.8}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The current $j_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}$ is sometimes called a "topological current." It is an abelian ($U(1)$ current) that is conserved

$$\partial^\mu j_\mu = -\epsilon_{\mu\nu\rho}\partial^\mu\partial^\nu A^\rho = 0 \ . \tag{1.9}$$

How does this current act? It is instructive to use the fact that a free photon and a free scalar are dual in 3D (in 4D, we have that a photon and a photon are dual)

$$\partial_\mu\phi = \epsilon_{\mu\nu\rho}F^{\nu\rho} \ . \tag{1.10}$$

Indeed, we have that $\partial^2\phi = 0$ is equivalent to $\epsilon_{\mu\nu\rho}\partial^\mu F^{\nu\rho} = 0$ and $\epsilon_{\mu\nu\rho}\partial^\nu\partial^\rho\phi \sim \partial^\nu F_{\mu\nu} = 0$.

• In a similar spirit, for a free massless chiral multiplet, we have that $\bar{\Phi}\Phi$ satisfies $\bar{\mathcal{D}}^2(\bar{\Phi}\Phi) = \mathcal{D}^2(\bar{\Phi}\Phi) = 0$. This is because, as you showed on the homework,

$$\bar{\mathcal{D}}^2(\bar{\Phi}\Phi) = (\bar{\mathcal{D}}^2\bar{\Phi})\Phi \sim (\epsilon^{\alpha\beta}\partial_{\bar{\theta}^\beta}\partial_{\bar{\theta}^\alpha}\bar{\Phi})\Phi \ , \tag{1.11}$$

where "$\sim$" denotes that we drop terms that are not zeroth order in the Grassmann variables. Therefore, we have

$$\bar{\mathcal{D}}^2(\bar{\Phi}\Phi) \sim \bar{F}\phi \ , \tag{1.12}$$

where we have again dropped higher-order terms in the Grassmann coordinates *and* neglected an overall non-zero constant. Now, if we go back to the Lagrangian of a free massless chiral superfield, we see that equations of motion for F imply

$$F = \bar{F} = 0 \ . \tag{1.13}$$

As a result, we have that, to lowest order,

$$\bar{\mathcal{D}}^2(\bar{\Phi}\Phi) = 0 \ . \tag{1.14}$$

Higher-order terms must also vanish: the SUSY variation of the lowest-order term is proportional to the $\mathcal{O}(\theta)$ and $\mathcal{O}(\bar{\theta})$ terms. But these terms vanish since the SUSY variation of zero is zero. Proceeding iteratively, we see that

$$\bar{\mathcal{D}}^2(\bar{\Phi}\Phi) = 0 \ . \tag{1.15}$$

Then, by computation, you can show that the $\theta\gamma^\mu\bar\theta$ component of $\bar\Phi\Phi$ is

$$j_\mu = i\phi\partial_\mu\bar\phi - i\bar\phi\partial_\mu\phi + \psi\gamma_\mu\bar\psi \ , \tag{1.16}$$

which is the Noether current for the $u(1)$ symmetry under which $\Phi\to e^{i\theta}\Phi$ and $\bar\Phi\to e^{-i\theta}\bar\Phi$.

• We can also write gauge invariant Lagrangians featuring these representations. For example, we have 3D $\mathcal{N}=2$ SQED with N_f flavors if we take the Lagrangian to be

$$\begin{aligned}
\mathcal{L}_{\rm SQED} &= -\int d^4\theta\sum_{i=1}^{N_f}(\bar q^i e^{2V}q_i + \bar{\tilde q}^i e^{-2V}\tilde q_i) - \frac{1}{g^2}\int d^2\theta d^2\bar\theta\Sigma^2 = \frac{1}{g^2}\Big(\frac12 D^2 - \frac14 F^{\mu\nu}F_{\mu\nu}\\
&- \partial^\mu\sigma\partial_\mu\sigma + i\lambda\gamma^\mu\partial_\mu\bar\lambda\Big) + \sum_i(|F_i|^2 - D_\mu\bar\rho^i D^\mu\rho_i + i\psi_i\gamma^\mu D_\mu\bar\psi^i + |\tilde F_i|^2 - D_\mu\bar{\tilde\rho}^i D^\mu\tilde\rho_i\\
&+ i\tilde\psi_i\gamma^\mu D_\mu\bar{\tilde\psi}^i - \sigma^2(|\rho_i|^2+|\tilde\rho_i|^2) - D(|\rho_i|^2 - |\tilde\rho_i|^2) - i\sigma(\psi_i\bar\psi^i - \tilde\psi_i\bar{\tilde\psi}^i\\
&- \sqrt2 i(\lambda\psi_i\bar\rho^i - \lambda\tilde\psi_i\bar{\tilde\rho}^i) - \sqrt2 i(\bar\lambda\bar\psi^i\rho_i - \bar\lambda\bar{\tilde\psi}^i\tilde\rho_i)) \ .
\end{aligned} \tag{1.17}$$

The flavor symmetry here is $U(1)^2\times SU(N_f)\times SU(N_f)$ (recall: this symmetry commutes with SUSY and the gauge symmetry)... Where one of the $U(1)$ factors is the topological symmetry. Recall that flavor symmetry—as opposed to R-symmetry—commutes with SUSY (both are generated by spin one currents and therefore have spin zero charges).

• The theory also has a $U(1)_R$ symmetry under which $R(\theta)=+1$, and, say, $R(\sigma)=0$, $R(\rho_i)=R(\tilde\rho_i)=\frac12$ (we fix the R charges of the rest of the fields in the multiplet by demanding that the superfield transforms with a single overall phase; the conjugate multiplets have opposite $U(1)_R$ charge).

• We would like to understand the $U(1)_R$ symmetry better. To do this, let's take a simpler theory first, the free massless chiral multiplet

$$\mathcal{L}_{\rm kin} = -\int d^4\theta\bar\Phi\Phi = |F|^2 - \partial_\mu\bar\phi\partial^\mu\phi + i\psi\gamma^\mu\partial_\mu\bar\psi \ . \tag{1.18}$$

This theory has a $U(1)$ flavor symmetry, $\mathcal{J}$ (with $j_\mu\in\bar\Phi\Phi$), which rotates each component field by the same phase, so $\mathcal{J}(\phi)=\mathcal{J}(\psi)=\mathcal{J}(F)=+1$ (e.g., $\phi\to e^{-i\alpha}\phi$, and oppositely for the conjugate fields).

• There is also a $U(1)_R$ symmetry under which $R(\theta)=+1$, $R(\phi)=\frac12$, $R(\psi)=-\frac12$, $R(F)=-\frac32$. Actually, we can deform $R_\kappa\to R+\kappa\mathcal{J}$ (with $\kappa\in\mathbb{R}$) and still have an R symmetry $R_\kappa(\theta)=+1$, $R_\kappa(\phi)=\frac12(1+2\kappa)$, $R(\psi)=-\frac12(1-2\kappa)$, $R(F)=-\frac32(1-\frac23\kappa)$.... Still, the one with $\kappa=0$, R_0, will turn out to be special as we will see.

Note: there are a finite number of *independent* symmetries (just two) here...

• The reason R_0 is special is that it is related to other symmetries the theory has. Which symmetries? For example, we can dilate spacetime as follows $x^\mu \to e^\lambda x^\mu$, where $\lambda \in \mathbb{R}$ (note that e^λ is not a phase). We say x^μ has scaling dimension -1. Then, we get

$$\begin{aligned} S &= \int d^3x \int d^4\theta \bar{\Phi}\Phi = \int d^3x \left(|F|^2 - \partial_\mu\bar{\phi}\partial^\mu\phi + \frac{i}{2}\left(\psi\gamma^\mu\partial_\mu\bar{\psi} - \partial_\mu\psi\gamma^\mu\bar{\psi}\right)\right) \\ &\to \int d^3x e^{3\lambda}\left(|F|^2 - e^{-2\lambda}\partial_\mu\bar{\phi}\partial^\mu\phi + e^{-\lambda}\frac{i}{2}\left(\psi\gamma^\mu\partial_\mu\bar{\psi} - \partial_\mu\psi\gamma^\mu\bar{\psi}\right)\right) \end{aligned} \tag{1.19}$$

To get a symmetry we should see if we can assign scaling transformations to the fields in order to make S invariant... How can we do this? Well, we can clearly take

$$\phi \to e^{-\frac{\lambda}{2}}\phi \ , \quad \bar{\phi} \to e^{-\frac{\lambda}{2}}\bar{\phi} \ , \quad \psi_\alpha \to e^{-\lambda}\psi_\alpha \ , \quad \bar{\psi}_\alpha \to e^{-\lambda}\bar{\psi}_\alpha \ , \quad F \to e^{-\frac{3\lambda}{2}}F \ , \quad \bar{F} \to e^{-\frac{3\lambda}{2}}\bar{F} \ . \tag{1.20}$$

This gives us a scaling symmetry since now S doesn't depend on λ... We say $\Delta(\phi) = \frac{1}{2}$, $\Delta(\psi) = 1$, and $\Delta(F) = \frac{3}{2}$. Since the theory is free, this is also true in the quantum theory. Note that we could have derived the above also from the superspace integral by noting that $d\theta \to e^{-\frac{\lambda}{2}}d\theta$.

• We say the above theory is scale invariant. Moreover, note that

$$R_0(\phi) = \frac{1}{2} = \Delta(\phi) \ . \tag{1.21}$$

This is not a coincidence as we will see.

• To understand why, we first take a detour and note that the theory has more symmetry: it is conformally invariant. This latter symmetry is equivalent to the existence of a traceless stress-tensor. In our theory, we have

$$\begin{aligned} T_{\mu\nu} &= -\eta_{\mu\nu}\mathcal{L} + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\partial_\nu\phi + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\bar{\phi})}\partial_\nu\bar{\phi} + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\psi_\alpha)}\partial_\nu\psi_\alpha + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\bar{\psi}_\alpha)}\partial_\nu\bar{\psi}_\alpha \\ &= \eta_{\mu\nu}\partial_\rho\bar{\phi}\partial^\rho\phi - \partial_\mu\bar{\phi}\partial_\nu\phi - \partial_\mu\phi\partial_\nu\bar{\phi} + \frac{i}{2}\partial_\nu\psi\gamma_\mu\bar{\psi} - \frac{i}{2}\psi\gamma_\mu\partial_\nu\bar{\psi} \ . \end{aligned} \tag{1.22}$$

Although the fermionic stress tensor is not symmetric, it can be improved (**Exercise**) so that

$$T'_{\mu\nu} = \eta_{\mu\nu}\partial_\rho\bar{\phi}\partial^\rho\phi - \partial_\mu\bar{\phi}\partial_\nu\phi - \partial_\mu\phi\partial_\nu\bar{\phi} + \frac{i}{4}\partial_{(\nu}\psi\gamma_{\mu)}\bar{\psi} - \frac{i}{4}\psi\gamma_{(\mu}\partial_{\nu)}\bar{\psi} \ . \tag{1.23}$$

However, this is not traceless, since

$$T'^{\mu}_{\mu} = \partial_\rho\bar{\phi}\partial^\rho\phi \ . \tag{1.24}$$

To make it traceless is easy

$$\tilde{T}_{\mu\nu} = T'_{\mu\nu} - \frac{1}{2}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)(\bar{\phi}\phi) \ . \tag{1.25}$$

The last term is an improvement term and now $\tilde{T}^\mu_\mu = 0$. Let us drop tilde from now on and take $\tilde{T}_{\mu\nu} \to T_{\mu\nu}$.

• It is now easy to construct a conserved dilation current

$$j^D_\mu = x^\nu T_{\mu\nu} \ , \quad \partial^\mu j_\mu = \eta^{\mu\nu} T_{\mu\nu} + x^\nu \partial^\mu T_{\mu\nu} = 0 \ . \tag{1.26}$$

Note this has the correct form to dilate space-time since we have $\delta x^\mu = -\epsilon x^\mu$, and $[-ix^\nu\partial_\nu, x^\mu] = -ix^\mu$.

• Since $T^\mu_\mu = 0$, we also have the following conserved currents

$$j_{\mu\nu} = 2x_\mu x^\rho T_{\rho\nu} - x^2 T_{\mu\nu} \ , \quad \partial^\nu j_{\mu\nu} = 2\delta^\nu_\mu x^\rho T_{\rho\nu} + 2x_\mu \eta^{\rho\nu} T_{\rho\nu} + 0 - 2x^\nu T_{\mu\nu} - 0 = 0 \ . \tag{1.27}$$

These may be more unfamiliar, but they give rise to so-called "special conformal" transformations, K_μ.

• The resulting algebra (in Lorentzian signature) is the conformal algebra and is isomorphic to $SO(3,2)$. This has ten generators: P_μ, $L_{\mu\nu}$, D, K_μ. They satisfy the following algebra:

$$\begin{aligned}
[D, P_\mu] &= iP_\mu \ , \quad [D, K_\mu] = -iK_\mu \ , \quad [D, L_{\mu\nu}] = 0 \ , \quad [K_\mu, P_\nu] = 2i(\eta_{\mu\nu}D - L_{\mu\nu}) \ , \\
[K_\rho, L_{\mu\nu}] &= i(\eta_{\rho\mu}K_\nu - \eta_{\rho\nu}K_\mu) \ , \quad [P_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu}P_\nu - \eta_{\rho\nu}P_\mu) \ , \\
[L_{\mu\nu}, L_{\rho\sigma}] &= i(\eta_{\nu\rho}L_{\mu\sigma} + \eta_{\mu\sigma}L_{\nu\rho} - \eta_{\mu\rho}L_{\nu\sigma} - \eta_{\nu\sigma}L_{\mu\rho}) \ .
\end{aligned} \tag{1.28}$$

• But we also have SUSY. So does this algebra close? Well, we know Super-Poincaré closes, but here we have introduced new generators. In particular we need something of dimension half from

$$[K_\mu, Q_\alpha] \sim \gamma_{\mu\alpha}{}^\beta S_\beta \ , \quad [K_\mu, \bar{Q}_\alpha] \sim \gamma_{\mu\alpha}{}^\beta \bar{S}_\beta \ . \tag{1.29}$$

These additional supercharges have dimension $-1/2$ (the corresponding current comes from a linear moment of the supercurrent). So, we need to add these two complex charges to the algebra. Then, we can check that

$$\{Q_\alpha, \bar{S}_\beta\} \sim \epsilon_{\alpha\beta} R + \cdots \ , \quad \{\bar{Q}_\alpha, S_\beta\} \sim -\epsilon_{\alpha\beta} R + \cdots \ , \tag{1.30}$$

where these relations contain additional charges in the ellipses (i.e., D and $L_{\mu\nu}$) and the relations hold up to overall constants (for more details, see [1,2]).

• In particular, we see that there is a "special" superconformal R symmetry that is related to the superconformal algebra. In the free case, this is exactly R_0.

• What does this mean more precisely (after all, non-R superconformal charges are invariant under flavor symmetries!!!)? Well, as we have argued above, conformal symmetry requires the existence of a conserved and *traceless* $T_{\mu\nu}$ (although, as we saw, there are generally families of stress tensors, and only one member is traceless). Now, note that the R-current is related to the stress tensor since

$$[Q_\alpha, R_{0;\mu}] = S_{\mu\alpha} \ \rightarrow \ \gamma_\nu^{\alpha\beta}\{\bar{Q}_\beta, [Q_\alpha, R_{0;\mu}]\} \sim T_{\mu\nu} \ . \tag{1.31}$$

Naturally, the superconformal $R_{0;\mu}$ is in the same multiplet as the *traceless* stress tensor (the only spin two $\theta\bar{\theta}$ component is the traceless stress tensor). Turning on a mixing with flavor currents relates it to a non-traceless stress tensor.

• **Exercise:** Check this statement by writing down the Noether current for $R_{0;\mu}$ and using SUSY variations of the chiral and anti-chiral superfields.

• How do we find the superconformal R symmetry from the set of all possible choices? Well, the superconformal R symmetry should commute with all continuous and discrete flavor symmetries as well (since it is part of an algebra whose elements all commute with these symmetries.... **Note:** this is a stronger statement than just saying that an R symmetry is an automorphism of an algebra). In the interacting examples we study this will be enough. **The above statement will be absolutely crucial in our interacting examples!!!** You will work out an example of this idea on the homework.

• While it is too advanced for our module, a more general and useful algorithm for finding the superconformal R symmetry in 3D $\mathcal{N} = 2$ is presented in [3] and further developed in [4].

• **Comment:** Since $R_0(\phi) = 1/2 = \Delta(\phi)$, we can read the scaling dimensions of ϕ^n from their R-charge. In this case, it gives the trivial result $R(\phi^n) = n/2 = \Delta(\phi^n)$. Usually, in quantum field theory you need to normal order to define composite operators (as you have seen in other modules).... For example, consider $\bar{\phi}\phi$... This needs to be normal ordered since

$$\langle\bar{\phi}(x)\phi(0)\rangle = \frac{1}{x} \ . \tag{1.32}$$

So, if we want to define $\lim_{x\to 0}\bar{\phi}(x)\phi(0)$, we need to subtract this divergence. That's exactly what normal ordering does... In the case of products of chiral operators (or products of anti-chiral operators), this is completely unnecessary (note $\langle\phi(x)\phi(0)\rangle = 0$)!

• This is more generally true in an interacting theory:

$$R_0(\mathcal{O}_i) = \Delta(\mathcal{O}_i) \ , \quad \bar{D}_\alpha \mathcal{O} = 0 \ . \tag{1.33}$$

and $R_0(\mathcal{O}_1\mathcal{O}_2) = \Delta(\mathcal{O}_1\mathcal{O}_2) = \Delta(\mathcal{O}_1) + \Delta(\mathcal{O}_2)$. So, these operators, as in QM, form a ring under the usual multiplication of operators... Proving this is beyond the scope of the course but follows from using the 2pt function to construct a norm $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle$ and demanding non-negativity of this norm (for the $\bar{Q}_\alpha$ descendant of a chiral operator... see [1] for more details)...

• **Comment:** Note that the superconformal R symmetry is a genuine symmetry of the theory as opposed to being just an automorphism of the algebra. For example, we could consider turning on

$$W = m\Phi^2 + \lambda\Phi^3 \ . \tag{1.34}$$

This theory doesn't have an R-symmetry (although the SUSY algebra has a $U(1)_R$ automorphism)... It can't because the integration measure $d^2\theta$ has $R = -2$, and we can't simultaneously have $R(\phi) = 1$ and $R(\phi) = 2/3$. It also doesn't have a superconformal R-symmetry... **Note:** We will study this theory in more detail in the next lecture...

• The above theory therefore cannot be both supersymmetric and conformal (the corresponding algebra does not close if we include both SUSY and conformal generators). Indeed, it is not conformal since W has scaling dimension two and so m has scaling dimension 1 and λ has scaling dimension 1/2. We will see soon using some fancy arguments that it is necessarily SUSY (i.e., has a SUSY ground state even in the quantum theory).

• Let us now understand non-conformal theories better. The simplest thing to do is to start with a free chiral multiplet and turn on

$$\delta W = m\Phi^2 \ . \tag{1.35}$$

This is a mass term. It also breaks the superconformal R symmetry since $R_0(\Phi^2) = 1 \neq 2$ (i.e., $R_0(m) = 1$). There is, however, an R-symmetry, $R_{\frac{1}{2}}$ (where $R_{\frac{1}{2}}(\Phi) = 1$).

• What does the propagator look like in this theory? Say we are in Euclidean space. Then, we have (up to an overall factor)

$$\langle \phi(k)\phi(-k)\rangle = \frac{1}{k^2 + m^2} \tag{1.36}$$

Clearly, in the limit $k \gg m$, we have that

$$\langle \phi(k)\phi(-k) \rangle \sim \frac{1}{k^2} \ . \tag{1.37}$$

This limit is called the UV (ultraviolet) or high energy / momentum limit. In this limit, the theory looks like a free massless scalar. On the other hand, for $k \ll m$, we are in the IR (infrared) or low energy / momentum limit. In this limit, we have that

$$\langle \phi(k)\phi(-k) \rangle \sim \frac{1}{m^2} \ , \tag{1.38}$$

and there is no energy to excite modes of the field...

• It is useful to Fourier transform the above to position space. In particular, we have $\langle \phi(x)\phi(y) \rangle$, and, for $|x-y| \ll m$ (the UV or short-distance limit), we have

$$\langle \phi(x)\phi(y) \rangle = \frac{1}{|x-y|} \ , \quad |x-y| \ll m \ . \tag{1.39}$$

On the other hand, in the IR (long-distance limit), we have

$$\langle \phi(x)\phi(y) \rangle = \delta^3(x-y) \ , \quad |x-y| \gg m \ . \tag{1.40}$$

The first limit is the CFT limit of the free scalar, while the second limit is clearly trivial (there is no propagation of fields...)... The theory is completely massive... In particular, in this regime, m provides a short-distance cut-off, so we never should consider the divergence when $x \to y$ (this divergence is an example of something called a local or "contact" term—it is related to the UV definition of the theory) **Exercise:** Perform the Fourier transform of the momentum-space 2-pt function and check that it interpolates between these two limits.

• Therefore, our theory interpolates between an SCFT in the UV and a trivial theory in the IR. To codify this, we introduce an energy momentum scale, μ—this is the scale at which we "observe" the theory... It is called the "RG scale"... When $\mu \to 0$ we go to the trivial theory and when $\mu \to \infty$ we get the UV CFT... To see the relative importance of couplings in the IR and UV, we define a dimensionless coupling

$$\hat{m} = \frac{m}{\mu} \ . \tag{1.41}$$

The importance of this coupling with scale is measured by the beta function

$$\beta_m \equiv \mu \frac{\partial \hat{m}}{\partial \mu} = -\hat{m} \ . \tag{1.42}$$

This means that as $\mu \to 0$, $\hat{m} \to \infty$ while for $\mu \to \infty$, $\hat{m} \to 0$... This is what we expect: in the UV limit, the masses are not important... The opposite is true in the IR.

• Note also, in case it wasn't clear, similar comments apply to the fermion. So, our RG flow is a flow between the free massless chiral multiplet SCFT and the trivial theory in the IR.

• This discussion shows that the set of ideas behind renormalization really have nothing to do in general with computing loop diagrams. In some cases, we may need to compute loop diagrams in order to compute beta functions, but the idea and utility of the RG is much greater than these particular applications.

• Next week, we will study interacting theories.

References

[1] S. Minwalla, *"Restrictions imposed by superconformal invariance on quantum field theories"*, Adv. Theor. Math. Phys. **2**, 783 (1998), `hep-th/9712074`

[2] B. Willett, *"Ph.D. Thesis"*, https://thesis.library.caltech.edu/7111/1/thesis.pdf ,

[3] D. L. Jafferis, *"The Exact Superconformal R-Symmetry Extremizes Z"*, JHEP **1205**, 159 (2012), `arXiv:1012.3210`

[4] C. Closset, T. T. Dumitrescu, G. Festuccia, Z. Komargodski & N. Seiberg, *"Contact Terms, Unitarity, and F-Maximization in Three-Dimensional Superconformal Theories"*, JHEP **1210**, 053 (2012), `arXiv:1205.4142`