SPA5304 Physical Dynamics Lecture 1

David Vegh

8 January 2019

1 Advanced Review of Newtonian Mechanics

1.1 Particle

Idealization for a small object.

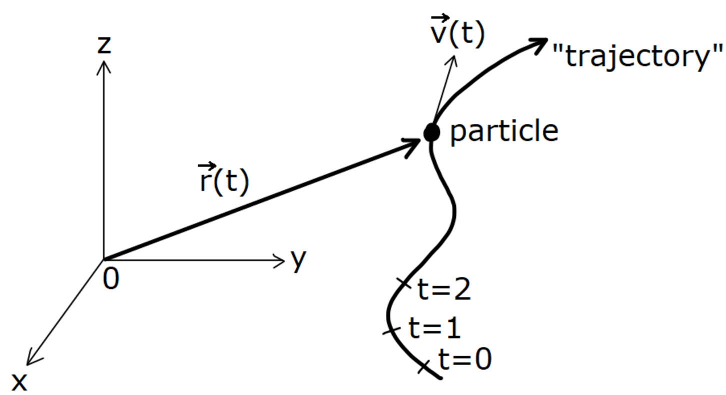
Properties:

- No size
- No internal motion (e.g. rotation)

Such an approximation is justified in suitable physical situations.

Cf. Rigid body: has finite size (but it can be described as a collection of particles).

1.2 Frame



- Need to introduce a **frame** to describe the motion of a particle (or a system of N particles, or a rigid body).
- The **position** of the particle is specified by **coordinates** (x, y, z) (sometimes denoted by x_1, x_2, x_3).

1.3 Trajectory

We want to describe the time-evolution of the system by obtaining the position of the particle at time t:

$$\vec{r}(t) = (x(t),\, y(t),\, z(t))$$

This is the trajectory of the particle.

(Sometimes we use $\vec{x}$ instead of $\vec{r}$.)

If the system consists of N particles, then use $\vec{r}_1(t)$, $\vec{r}_2(t), \ldots, \vec{r}_N(t)$.

1.4 Velocity

$$\vec{v} \equiv \frac{d\vec{r}}{dt} \equiv \dot{\vec{r}} \qquad \text{tangent to trajectory at time t}$$

Here $\cdot \equiv \frac{d}{dt}$ denotes time derivative.

$|\vec{v}(t)|$ is the **speed** at time t.

1.5 Acceleration

$$\vec{a}(t) \equiv \frac{d\vec{v}}{dt} \equiv \dot{\vec{v}} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}}$$

1.6 Facts

- Space is 3-dimensional: x, y, z
- Time is 1-dimensional: t

1.7 Newton's Principle of Relativity

There exists a coordinate frame, called **inertial frame**, in which

- Laws of physics are the same at all times.
- Another frame in uniform straight-line (rectilinear) motion w.r.t. an inertial frame is also inertial.

We will mostly use inertial frames.

1.8 Newton's Principle of Determinacy

If, at $t = t_0$, one is given positions $\vec{r}(t)$ and velocities $\vec{v}(t)$, and one knows the **forces** $\vec{F}(t)$ acting on the particles,

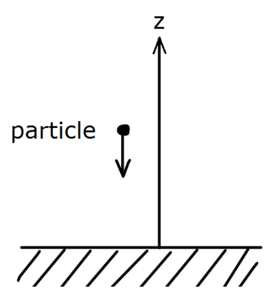
$\Rightarrow$ then subsequent motion for $t > t_0$ is completely determined.

How? . . . By Newton's 2nd law

$$\boxed{m\ddot{\vec{r}}(t) = \vec{F}(\vec{r},\, \dot{\vec{r}},\, t)}$$

Forces can depend on positions, velocities, and time.

1.8.1 Example: falling particle



Newton's 2nd law:

$$m\ddot{\vec{r}}(t) = -mg\hat{z} \qquad \text{(here } \hat{z} \text{ is the unit vector along the positive z-axis)}$$

This equation can be integrated to find the particle motion. It is a second order differential equation, therefore there will be two constants of integration (i.e. initial position and velocity).

1.9 Number of degrees of freedom and generalized coordinates

System of N particles

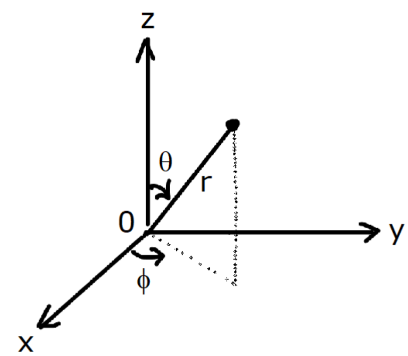
$$\left.\begin{array}{c} \vec{r}_1 = (x_1,\, y_1,\, z_1) \\ \vdots \\ \vec{r}_N = (x_N, y_N, z_N) \end{array}\right\} \; 3N \text{ coordinates}$$

The $3N$ Cartesian coordinates may not be convenient for describing the system in question.

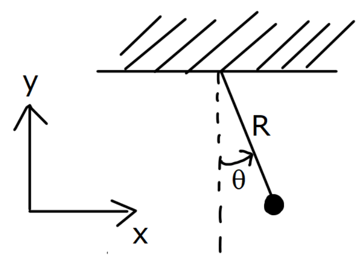
- Curvilinear coordinates may be more suitable than Cartesian ones.
- Physical conditions may reduce the number of independent coordinates.

1.9.1 Example: Spherical polar coordinate system $(r,\, \theta,\, \phi)$

Useful when there is a central force.



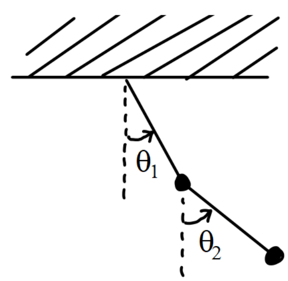
1.9.2 Example: Pendulum oscillating in a plane ("planar pendulum")



$$\vec{r} = R\,(\sin\theta,\, -\cos\theta,\, 0)$$

One quantity θ determines *three* Cartesian coordinates $\Rightarrow$ # of **degrees of freedom** (DoF) is 1.

1.9.3 Example: Coplanar double pendulum



Two quantities θ_1, θ_2 determine *six* Cartesian coordinates $\quad \Rightarrow \quad$ #Dof $= 2$.

1.10 Number of degrees of freedom

The number of independent quantities that must be specified in order to uniquely define the position of a system.

1.11 Generalized coordinates

A set of quantities that completely defines the position of a system.

If # of DoFs is s (where $s \leq 3N$), then the generalized coordinates can be written as

$$\vec{q} = (q_1, \ldots, q_s)$$

Generalized velocities:

$$\dot{\vec{q}} = (\dot{q}_1, \ldots, \dot{q}_s)$$

In terms of $\vec{q}$, the position in Cartesian coordinates can also be expressed

$$\vec{r}_1 = \vec{r}_1(\vec{q}, t) \tag{1}$$
$$\vdots \tag{2}$$
$$\vec{r}_N = \vec{r}_N(\vec{q}, t) \tag{3}$$

We will see many examples of generalized coordinates later.

Now let's go back into Cartesian coordinates. . .

1.12 Momentum

$$\vec{p} \equiv m\vec{v} = m\dot{\vec{r}} \qquad \text{(non-relativistic)}$$

1.13 Newton's 2nd law (single particle)

Equation of motion:

$$\boxed{\dot{\vec{p}} = \vec{F}}$$

$\vec{F}$ can be a sum of forces.

$$\dot{\vec{p}} = \frac{d}{dt}(m\vec{v}) = m\vec{a} + \underbrace{\dot{m}\vec{v}}_{\substack{\text{relevant in e.g.} \\ \text{rocket science}}}$$

If $\dot{m} = 0$ (as we assume), then

$$\vec{F} = m\vec{a} = m\ddot{\vec{r}}$$

1.14 Conservation of momentum

If the total force on the particle is zero ($\vec{F} = 0$), then $\dot{\vec{p}} = 0$.

This means that

$$\boxed{\vec{p} = \text{const.}}$$

i.e. momentum is conserved.

This is the first example of a **conservation law**.

We will encounter other conservation laws later.